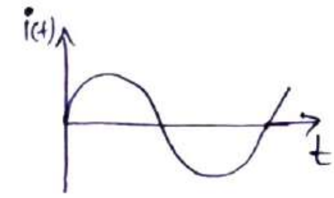
UNIT - II

A.C Circuits

Alternating Quantity : "A quantity which changes periodically its magnitude and direction with respect to time" is known as "Alternating quantity".

Alternating Current : (A.C) : "The current which changes periodically its magnitude and direction with respect to time, is called as "Alternating Current".

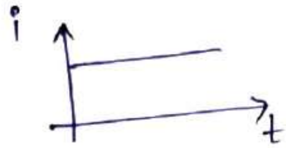
Ex :-

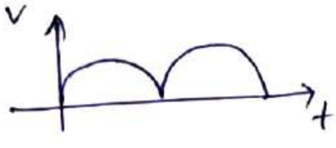


- it has magnitude & phase angle
- it has finite frequency
- it is dependent of the time
- it changes direction in +Ve & -Ve field.

Direct Current (D.C) : The current which maintains constant magnitude & only one direction

Ex :-





- It is independent of the time.
- frequency is "zero (0)
- Magnitude is constant.

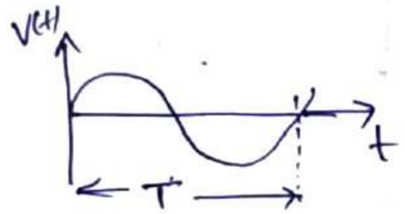
Cycle : A set of positive & negative instantaneous volue of the alternating quantity.

$$1 \text{ Cycle} = 2\pi \text{ radians (or) } 360^\circ.$$

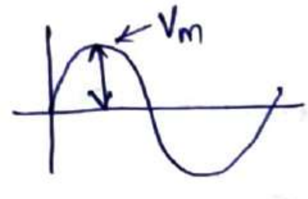
Frequency : (f) : Frequency is "no. of cycle per a second" for an alternating quantity.

$$\boxed{f = \frac{1}{T}}$$ units "Hz" (Hertz)

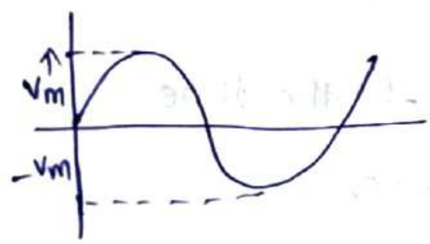
Time period (T) : It the time required by the alternating quantity to complete one cycle"



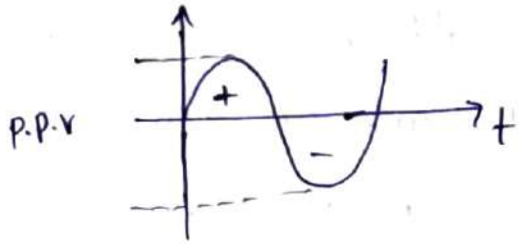
Maximum Volue :- The maximum Value is the "Value reached by alternating quantity in total time 't'"



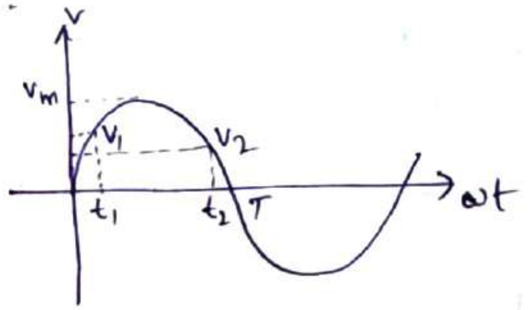
Peak Value :- It is the maximum Value of the wave during either positive (or) negative half cycle



Peak to Peak Value :- It is maximum difference between peak to peak values of +ve and -ve cycles.



Instantaneous Value :- The value of alternating quantity at a particular instant time is known as instanteneous value.



where
V_1 is I.V of wave at t_1
V_2 is I.V of wave at t_2.

Angular frequency (ω) :-

It is the frequency expressed in electrical radians per second.

It is expressed as $\Rightarrow 2\pi \times \text{Cycle/sec}$

i.e $\boxed{\omega = 2\pi f}$ rad/sec (or) $\boxed{\omega = \frac{2\pi}{T}}$ rad/sec

& $\boxed{\theta = \omega t}$ rad (or) $\boxed{\theta = 2\pi f t}$ rad.

Equation of Alternating quantity :-

For AC - Current $i(t) = I_m \sin \omega t$ (or) $i(t) = I_m \sin \theta$.

where $I_m \rightarrow$ maximum current
$\omega \rightarrow$ angular frequency.

R.M.S Value (Root Mean Square Value)

(or) Effective Value

The RMS value of an alternating current is equalent to steady current (D.C-current), which produces the same amount of heat as that produced by an alternating current when it passing through a same circuit for a same time (or) specified time.

(or)

The RMS-value of an alternating quantity means that square root of average of the squares of it's instanteneous values over a one-complete cycle.

→ For any alternating quantity $f(t)$, the time period 'T' then rms value is

$$f_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\, dt} \Rightarrow \sqrt{\frac{\text{Area covered by wave}}{\text{Length of base over a cycle}}}$$

→ If $f(t) = (a_0 + a_1 \cos\omega t + a_2 \cos 2\omega t + \ldots) + (b_1 \sin\omega t + b_2 \sin 2\omega t) + \ldots)$

then

$$f_{rms} = \sqrt{a_0^2 + \frac{a_1^2 + a_2^2}{2} + \ldots \frac{b_1^2 + b^2}{2} \ldots}$$

i.e $$f_{rms} = \sqrt{\frac{f_1^2 + f_2^2 + f_3^2 \ldots f_n^2}{n}}$$

Note:- For Alternating Current RMS value can be calculated for complete one cycle.

Average Value :-

The Average Value can be defined as the direct steady current (D.C) which transfer charge across any circuit, the same amount of charge transfered by that alternating current during the same time for the Same time

(or)

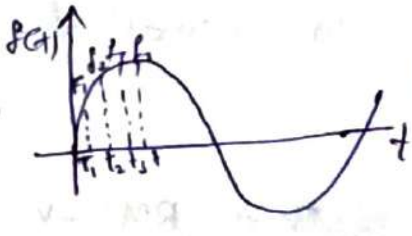
The Average Value of an alternating quantity is defined as that value which is obtained averaging all the instanteneous values over a period of half cycle.

i.e $$f_{avg} = \frac{1}{T}\int_0^T f(t)\,dt$$

at $T = \frac{2\pi}{2} = \pi$

(or) $$f_{avg} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}$$

(or) $$f_{avg} = \frac{f_1 + f_2 + f_3 + \dots f_n}{n}$$ $\Rightarrow$



Note :- Average value for AC-Signal is zero, for that we have to calculate averge value for half cycle.

Form Factor :- Form factor is the ratio of RMS & Average Values

i.e Form factor (F.F) $= K_f = \frac{\text{RMS Value}}{\text{Averge value.}}$

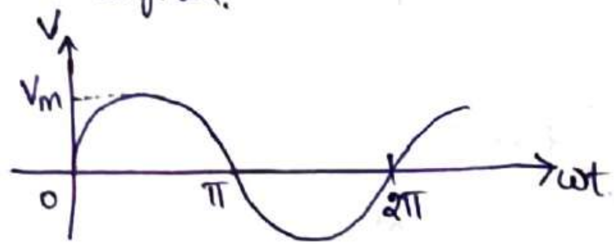
Peak factor (or) Crest factor

Peak factor it is the ratio of Peak value (or) maximum value to the RMS value

i.e $P.F = \dfrac{\text{Maximum (or) Peak value}}{\text{RMS value.}}$

Problem:- Determine RMS value for given signal

(or) Derive the relation between RMS value & maximum value. for given signal.



Solution:- For given signal is Sinusoidal Alternating voltage, its mathematical equation is

$$V = V_m \sin\omega t \quad \text{with timeperiod } T = 2\pi$$

Then RMS-value is

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V^2\, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\omega t)^2\, d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\left[\frac{1-\cos 2\omega t}{2}\right] d(\omega t)}$$

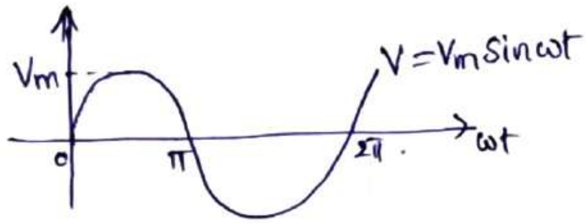
$$= \sqrt{\frac{V_m^2}{2\pi}\cdot\frac{1}{2}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\cancel{\pi}} \cdot \frac{1}{\cancel{2}}\left[\cancel{2\pi} - 0\right]}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \Rightarrow \boxed{V_{rms} = 0.707\, V_m}$$

Problem:- Determine Average Value for given wave form

(or) Derive the relation between Average Value & maximum Value



Solution:- For given Alternating voltage wave form, Average Value is zero for "total timeperiod i.e 2π

So, let half time period as "T = π" . then

$$V_{avg} = \frac{1}{T}\int_0^T V \, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin(\omega t)\, d(\omega t)$$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$= \frac{V_m}{\pi} \cdot \left[-\left[\cos\pi - \cos 0\right]\right]$$

$$= \frac{V_m}{\pi} \cdot \left[-\left[-1 - 1\right]\right]$$

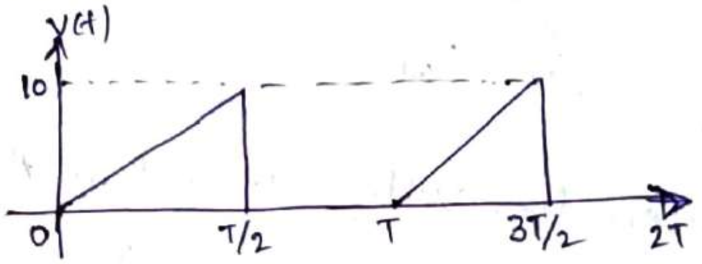
$$V_{avg} = \frac{2V_m}{\pi} \longrightarrow V_{avg} = \frac{2V_m}{\pi} \Rightarrow \boxed{V_{avg} = 0.637\, V_m}$$

For Sinusoidal Signal Form factor & peak factor are

$$\text{Form factor} = \frac{\text{RMS Value}}{\text{Avg Value}} = \frac{0.707\, V_m}{0.637\, V_m} = 1.11$$

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{rms Value}} = \frac{V_m}{0.707\, V_m} = 1.414$$

Problem:- Determine RMS & Avg Values, form factor & peak factor of periodic function



Solution:- From Given data, we have to calculate function equation with in the limits of $0 < t < T$

i.e

$0 < t < T/2$ & $T/2 < t < T$

so

v(t) points are $(0,0), (T/2, 10)$ $(T/2, 0)\ (T, 0)$

(x, y)
$(v(t), t)$ $\Rightarrow v(t) - 0 = \frac{10}{T/2}(t - 0)$ $v(t) - 0 = \frac{0-0}{T-T/2}(t - T/2)$

$v(t) = \frac{20}{T} t$ $v(t) = 0.$

Now RMS Value is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{T/2} \left(\frac{20}{T} t\right)^2 dt}$$ $\because \int_{T/2}^{T} v(t)\, dt = 0$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T/2} \frac{400}{T^2}\, t^2\, dt}$$

$$= \sqrt{\frac{400}{T^3} \cdot \cancel{400}\left(\frac{t^3}{3}\right)_0^{T/2}}$$

$$= \sqrt{\frac{400}{3.\cancel{T^3}}\left(\frac{\cancel{T^3}}{8} - 0\right)} = \sqrt{\frac{400}{24}} \Rightarrow \boxed{V_{rms} = 4.08}$$

Average Value:

$$V_{Avg} = \frac{1}{T}\int_0^T v(t)\, dt$$

$$= \frac{1}{T}\int_0^{T/2} \frac{20}{T} t\, dt + \underbrace{\int_{T/2}^{T} \frac{-20}{T} t\, dt}_{= 0 \text{ (zero)}}$$

$$= \frac{20}{T^2}\int_0^{T/2} t\, dt$$

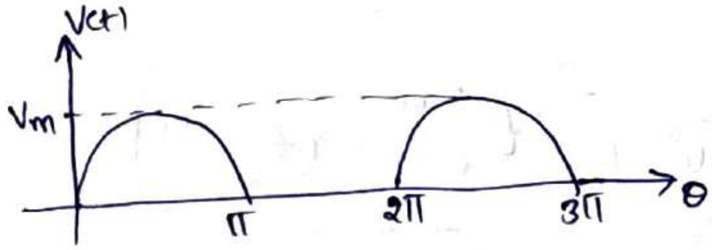
$$= \frac{20}{T^2}\left[\frac{t^2}{2}\right]_0^{T/2} = \frac{20}{2\cancel{T^2}} \cdot \frac{\cancel{T^2}}{4} \Rightarrow \frac{20}{8} \Rightarrow V_{avg} = 2.5$$

Then

$$\text{Form factor } F.F = \frac{\text{rms Value}}{\text{Avg Value}} = \frac{4.08}{2.5} = 1.632$$

$$\text{Peak Value } P.F = \frac{\text{Maximum Value}}{\text{Rms Value}} = \frac{10}{4.08} = 2.45$$

HW Determine the rms & Averge Values of a half wave rectified sinusoidal voltage of a peak value Vm



Ans:- $V_{rms} = V_m/2$

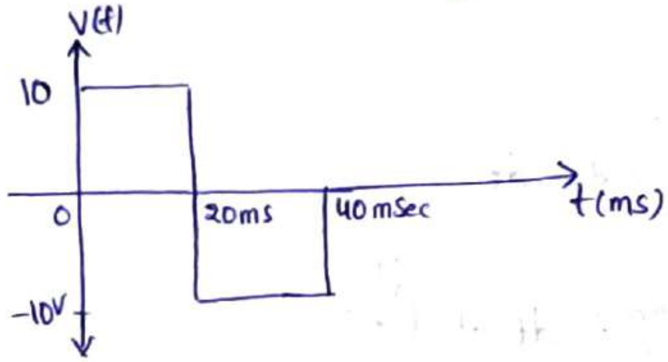
$V_{avg} = V_m/\pi$.

problem:-

Obtain RMS-Value, Average Value, Form factor & peak Value for a voltage of Symmetrical Square shape whose amplitude is 10 V and time period is 40 msec

Solution:-

From the given data Voltage wave form can be drawn as



For total time period of wave is 40 msec, then Voltage equation for different intervals

for $0 < t < 20ms \rightarrow V(t) = 10V$

$20ms < t < 40ms \rightarrow V(t) = -10V$

For RMS Value:-

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V(t))^2 dt}$$

then

$$V_{rms} = \sqrt{\frac{1}{40ms}\left[\int_0^{20ms} (V(t))^2 dt + \int_{20ms}^{40ms} (V(t))^2 dt\right]}$$

$$= \sqrt{\frac{1}{40ms}\left[\int_0^{20ms} (10)^2 dt + \int_{20ms}^{40ms} (-10)^2 dt\right]}$$

$$= \sqrt{\frac{100}{40\times 10^{-3}}\left[(t)_0^{20ms} + (t)_{20m}^{40ms}\right]}$$

$$= \sqrt{\frac{100}{40ms}\left[20ms - 0 + 40ms - 20ms\right]}$$

$$V_{rms} = \sqrt{100} = 10V$$

For Average Value

$$V_{avg} = \frac{1}{T/2} \int_0^{T/2} v(t)\, dt$$

$$= \frac{1}{20ms} \int_0^{20ms} 10\, dt$$

$$= \frac{10}{20ms} (t)_0^{20m} \Rightarrow \frac{10}{20ms}(20ms - 0) \Rightarrow \boxed{V_{avg} = 10}$$

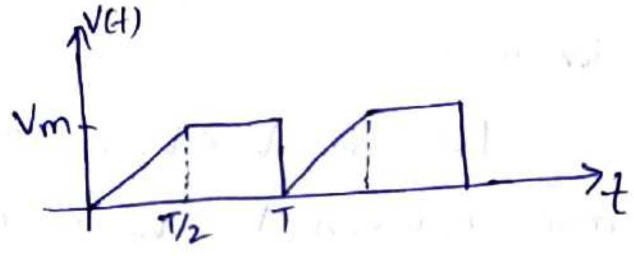
$$\text{Form Factor} = \frac{V_{rms}}{V_{avg}} = \frac{10}{10} = 1$$

$$\text{Peak factor} = \frac{V_p}{V_{rms}} = \frac{10}{10} = 1$$

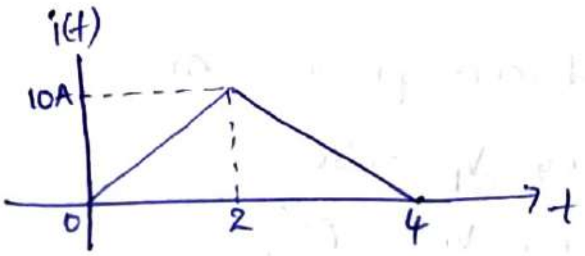
Q:- Find Average, RMS, Form factor & peak factor for given voltage wave form which is having 20V maximum Value at $0 < t < 0.1$ sec, $0.3 < t < 0.4$ intervals and at the "0V" the interval of $0.1 < t < 0.3$ sec.

Q:- Determine Avg, RMS, F.F & P.F for given signals

(i)



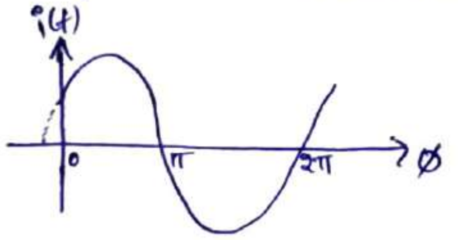
(ii)

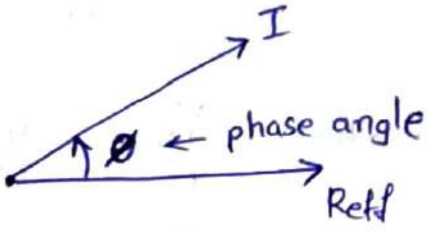


Phase :- Phase is the relative position of the waveform with respect to zero position.

(or)

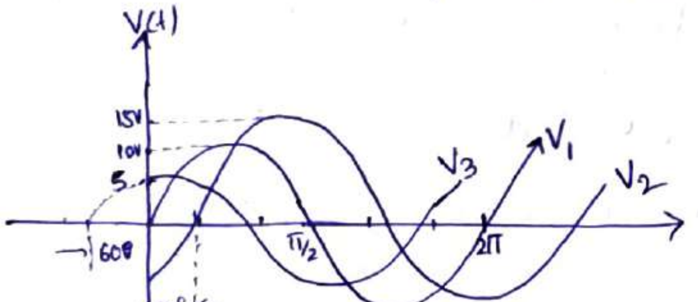
Phase is the relative position of the phase or phasor with respect to reference phase



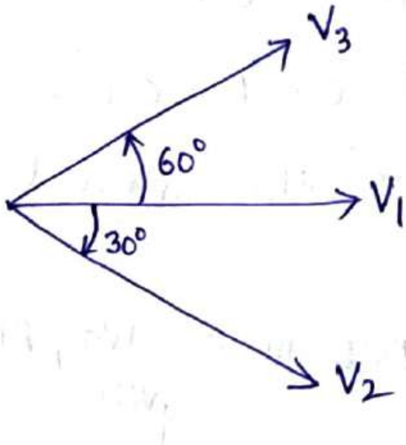


Vector diagram.

Phase difference :- It is the phase angle difference between two phasors or vectors



⇒



From the above vector diagram

We have phase difference between V_1 & V_2 is 30°

we have phase difference between V_1 & V_3 is 60°

And

if we take `V_1' is reference phasor then

V_2 is lags by $V_1 \rightarrow 30^\circ$

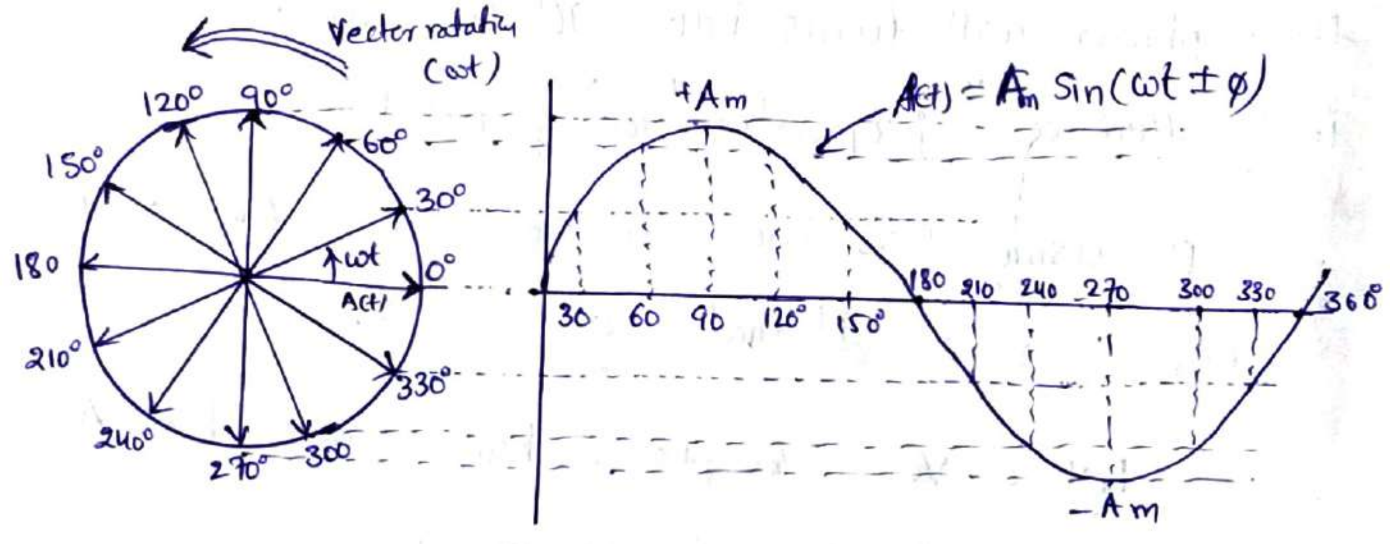
V_3 is lead by $V_1 \rightarrow 60^\circ$.

Phasor representation of an Alternating Quantity :-

The graphical reprecention of sinusoidally varying alternating quantity by a straight line with an arrow, such line is called as "phasor" or "Vector"

The diagram which represents the different alternating quantities by individual phasors, which gives the exact phasor interrelationships is known as phasor diagram.

Let phasors are assumed to be rotated in anticlockwise direction with constant speed, as shown below



→ consider a phasor rotating in anticlockwise direction from the phasor angle 0° to 30°, then alternating quantity increases from 0 to upto 30° on y-axis

→ And from 30° to 60° phasor rotates then again alternating quantity function will changes. as

i.e $A(t) = A_m \sin\theta$

as like that

at 90° it is $A(t) = A_m$

& 180° it is $A(t) = 0$

& 210 ~~or~~ 240 it is $A(t) = -A_m$ ... and so on,

the alternating quantity can be changed.

Like ~~thing~~ this type of rotating vector is called phasor, and this diagram is called as " phasor diagram ".

J- operator or j-notation :-

It is a quantity when it acts on any phasor then phasor will turns into 90° in anticlockwise direction it is called as " j-operator " or ' j- notation '.

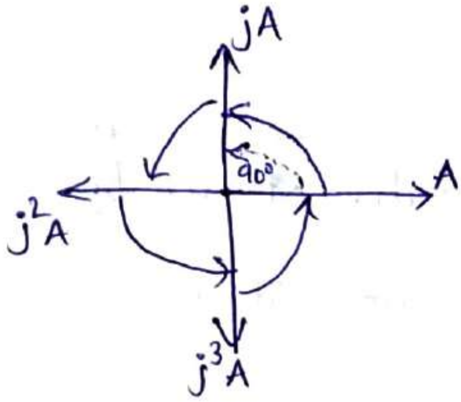
By using this we can solve the complex phasor calculations of alternating quantities.

Let a vector or phasor having magnitude A & phase angle 0°, then it shown as

$A\angle 0^\circ$

if we apply j-operator then it rotates to 90° anticlockwise direction

i.e $A\angle 90^\circ$



$A\angle 0$
to ↓
$A\angle 90$
to ↓
$A\angle 180°$
↓
$\angle 270°$

as
$j = \sqrt{-1} = 1\angle 90°$
$j^2 = -1 = 1\angle 180°$
$j^3 = -j = 1\angle 270$
$j^4 = 1 = 1\angle 360°$

Repreentation of Altenating Quantities :- It is by two forms,

① Reetangulor form

It is the form is
Sum of real & imajinary parts

i.e $z = a + jb$

→ It is usefull fur
adding & Substraction of A.Q's

Add → $(R_1+R_2)+j(x_1+x_2)$

Sub → $(R_1-R_2)+j(x_1-x_2)$

⇒ Reetangulor to polar conversion is

$z = a+jb \Rightarrow r\angle\theta$

then
$r = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}(b/a)$

② Polar form

it is reprecents
as magnitude & phase angle

$p = r\angle\theta°$

→ It usefull for
Multiplaying & dividing

Multply → $r_1 r_2 \angle\theta_1+\theta_2$

devidin → $\frac{r_1}{r_2}\angle\theta_1-\theta_2$

⇒ polar to Rectangular form

$r\angle\theta \Rightarrow z = a+jb$

$a = r\cos\theta$

$b = r\sin\theta$

Basic definitions :-

Impedance (Z) : It is the total opposition offered by an AC-circuit for the flow of current through it is called "Impedance"

(or)

The ratio of phasor voltage (V) to the phasor current (I) is called "Impedance"

i.e $Z = R \pm jX$ (or) $Z = \frac{V\angle\theta_1}{I\angle\theta_2}$

→ It magnitude value is $Z = \sqrt{R^2+X^2}$

→ Units are "Ω"

Reactance (X) : The opposition offered by reactive components i.e inductor (or) capacitor in an A.C circuit for the flow of current through it is called "Reactance(X)"

i.e imajinary part of impedance

$Z = R \pm jX$

for inductor it is "X_L" & for capacitor it is "X_C"

X_L = inductive reactance

X_C = capacitive reactance

$$\boxed{X_L = 2\pi f L}$$

$$\boxed{X_C = \frac{1}{2\pi f C}}$$

→ Units are "Ω"!

Admittance :- (Y) : The reciprocal of impedance is called as Admittance (Y).

→ Units are ℧ (mho)

i.e $Y = \frac{1}{Z} = \frac{1}{R \pm jX}$

by Rationalizing the denominator we get

$$Y = \frac{R}{R^2+X^2} \mp \frac{jX}{R^2+X^2}$$

$$\boxed{Y = G \mp jB}$$

Where G → Conductance $= \frac{R}{R^2+X^2}$

B → Susceptance $= \mp \frac{jX}{R^2+X^2}$

Susceptance (B) : The imajinary part of admittance is called as susceptance (B)

→ Units are "Mho(℧) (or) Siemens

$$B = \mp \frac{jX}{R^2+X^2}$$

where $\frac{-X}{R^2+X^2}$ → inductive Susceptance

$\frac{+X}{R^2+X^2}$ → Capacitive Susceptance

Conductance (G) :- It is the real part of admittance

$G = \frac{R}{R^2+X^2}$ units are "℧".

Real Power (or) Active Power (or) True Power (or) Average Power (or) Watt Power (P)

The product of r.m.s values of voltage and current with the cosine of angle between them is called "Active Power".

i.e $\boxed{P = VI \cos \phi}$

→ Units are Watts.

(or)

The power in a.c circuit can be defined as the product of the r.m.s values of voltage and ~~current with the sine of angle between them is called~~ active component of the current is called "Active power".

Reactive Power (or) Wattless Power (or) Quadrature Power (Q)

The product of r.m.s values of voltage & current with the sine of angle between them is called "Reactive power"

i.e $\boxed{Q = VI \sin \phi}$

→ Units are VAR – Volts Amperes per ~~Reactive~~ Reactance

(or)

The power in a.c circuit can be defined as the product of r.m.s value of voltage and reactive component of the current is called "Reactive Power Q".

Apparent Power (S)

The product of r.m.s values of voltage & current is called "Apparent Power" in a.c circuit

i.e $S = V_{rms} \times I_{rms}$

(or) $S = \sqrt{P^2 + Q^2}$ (or) $S = P \pm jQ$

Power factor :-

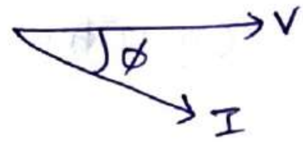
Power factor is the "cosine of the angle between voltage phasor to current phasor in a.c circuit.

The term "$\cos\phi$" is called power factor.

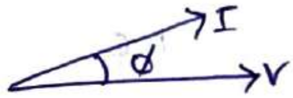
Example :-

for Resistive load → $\cos\phi = 1$ (Inphase)

Inductive load → Current lags



Capacitive load → Current leads



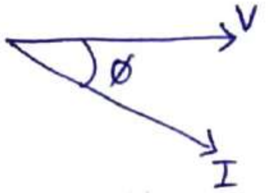
Power Triangle :- Power triangle is the geomentrical reprecentation of the apparent power, active power & reactive power.

We have power Triangle for → Inductive load
→ Capacitive load.

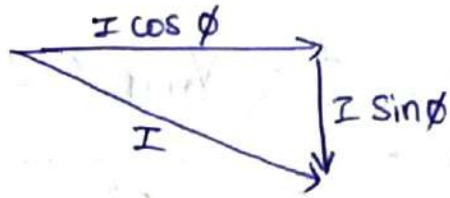
Power Triangle for Inductive load:-

For inductive load current laggs by Voltage

i.e



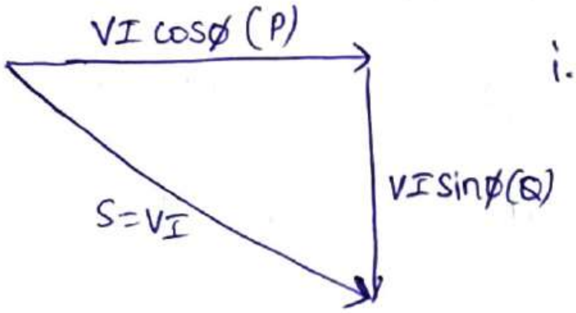
So



$I\cos\phi$ = active component

$I\sin\phi$ = reactive component

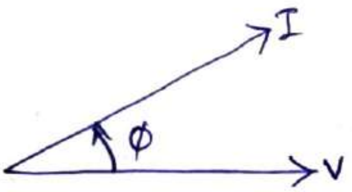
the power triangle is

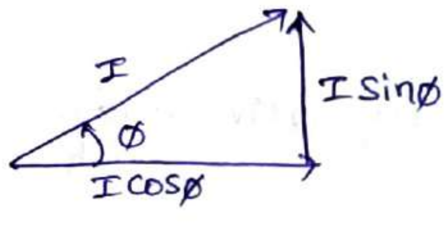


i.e
$$S = \sqrt{P^2+Q^2}$$
$$S = \sqrt{(VI\cos\phi)^2+(VI\sin\phi)^2}$$
$$S = VI$$

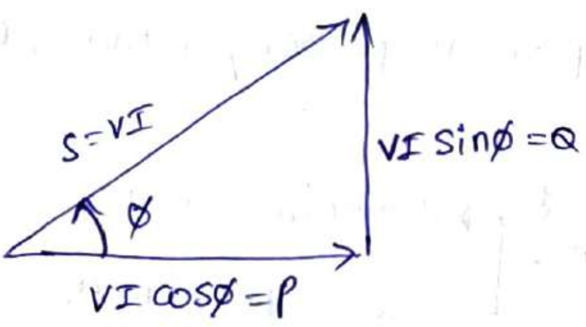
Power Triangle for Capacitive load:-

For capacitive load current leads by voltage





Power Triangle is



$$S = \sqrt{P^2+Q^2}$$
$$S = VI.$$

Complex power :-

Complex power is the product of voltage to the conjugate current component (or) product of conjugate voltage component to current

$$C.P = VI^* \text{ (or) } V^*I$$

Let
$$VI^* = Ve^{j\theta} . Ie^{-j(\theta+\phi)}$$
$$= VI e^{-j\phi}$$

$$VI^* = VI\cos\phi - j\,VI\sin\phi$$

$$S = \boxed{VI^* = P - jQ}$$ — is for Capacitive Circuit

Simillarly for inductive

$$\boxed{S = V^*I = P + jQ}$$ ← is for inductive Circuit.

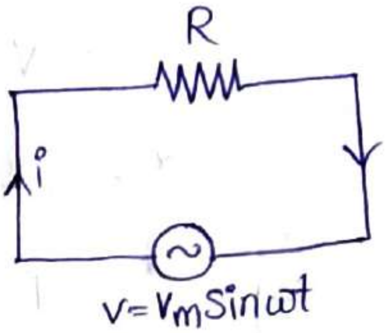
Analysis of Single-phase ac-circuits

AC through Pure Resistance :-

(or)

Sinusoidal response of Pure Resistive Circuit

consider a simple circuit consisting of a pure resistance 'R' connected with sinusoidal voltage $V = V_m \sin\omega t$, as shown below



→ According to ohm's the current in circuit is

$$i = \frac{V}{R} = \frac{V_m \sin\omega t}{R}$$

i.e $i = \left(\frac{V_m}{R}\right) \sin\omega t$ ⟵ ①

→ compare eq ① with standard current equation

$$i = I_m \sin(\omega t \pm \phi)$$

then $I_m = \frac{V_m}{R}$ & $\phi = 0°$

→ The maximum current value is $I_m = \frac{V_m}{R}$

→ $\phi = 0$ reprecent voltage & currents are inphase so

$$i = I_m \sin\omega t \quad \text{at } \phi = 0$$

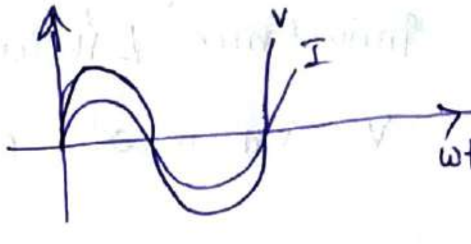
→ The instantaneous power is product of instantaneous values of voltage and current

$$P_i = v \times i$$

$$= V_m \sin\omega t \times I_m \sin\omega t$$

$$= \frac{V_m I_m}{2} \sin^2\omega t$$

$$= \frac{V_m I_m}{2} (1 - \cos 2\omega t)$$



$$P_{inst} = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos(2\omega t) \quad \leftarrow (2)$$

In eq (2) the term $\frac{V_m I_m}{2}\cos(2\omega t)$ is having double frequency

So this value becomes zero,

→ The average power value is equal to instantaneous power of constant power component $\frac{V_m I_m}{2}$

$$P_{avg} = \frac{V_m I_m}{2}$$

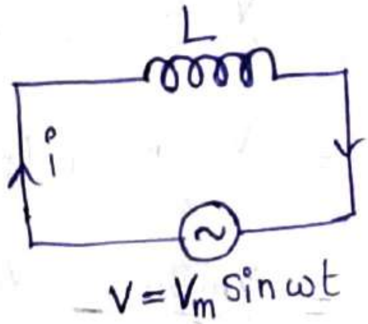
$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$\boxed{P_{avg} = V_{rms} \cdot I_{rms}}$$

For pure resistive circuit power is $\boxed{P = VI}$ Watts.

AC - through Pure Inductance

consider a simple circuit consisting of a pure inductance L then connected with sinusoidal voltage $v = V_m \sin\omega t$ as shown below.



For pure inductive circuit current laggs by voltage 90°, it can be proved as

Voltage across inductor is $V = L\frac{di}{dt}$

So $$V_m \sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin\omega t$$

$$i = \int di = \int \frac{V_m}{L}\sin\omega t$$

$$= \frac{V_m}{L} \cdot \frac{-\cos\omega t}{\omega}$$

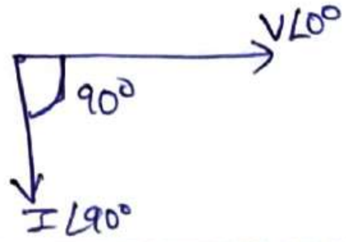
$$i = \frac{V_m}{\omega L}\sin(\omega t - \frac{\pi}{2})$$

& we have standard eqn $i = I_m \sin(\omega t \pm \phi)$

$i = I_m \sin(\omega t - \frac{\pi}{2})$ where $I_m = \frac{V_m}{X_L}$ & $\phi = -\frac{\pi}{2}$

Above equation gives that current lags by voltage 90°

vector diagram is



→ The instantaneous power

$$P_{inst} = v \times i = V_m \sin\omega t \times I_m \sin(\omega t - \tfrac{\pi}{2})$$

$$= -V_m I_m \sin\omega t \cos\omega t \qquad \because \sin 2A = 2 \sin A \cos A$$

$$P_{int} = \frac{-V_m I_m}{2} \sin(2\omega t)$$

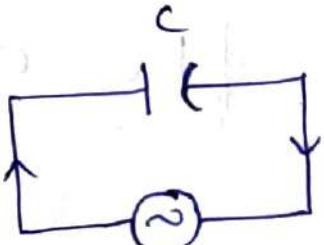
→ The Average power Value for pure inductive is zero, why because we have double frequency term ($\sin 2\omega t$)

so $\boxed{P_{avg} = 0}$

Pure inductance never consumes power

AC- through Pure Capacitance

consider a simple circuit consisting of a pure capacitan C Farady connected with sinusoidal voltage $V = V_m \sin\omega t$.



For pure capacitive circuit current leads by Voltage $90 = \pi/2$, it can be proved as

→ The instantaneous charge in capacitor is

$$q = C.V = C.V_m \sin\omega t.$$

→ Current is is rate of charge so $i = \frac{dq}{dt}$

$$i = \frac{d}{dt}(C\,V_m \sin\omega t)$$

by standard current Equation : $i = I_m \sin(\omega t \pm \phi)$

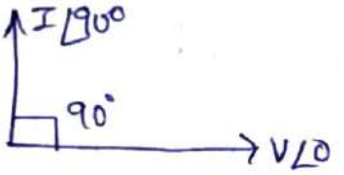
$$= C \cdot V_m \cdot (\omega \cos\omega t)$$

the

$$i = \left(\frac{V_m}{1/\omega C}\right) \sin\left(\omega t + \frac{\pi}{2}\right) \rightarrow I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

The equation give that Current leads by voltage $90° = \pi/2$

i.e



$$\phi = +\frac{\pi}{2}.$$

→ The instantaneous power

$$P_{inst} = V \times i = V_m \sin\omega t \times I_m \sin(\omega t + \pi/2)$$

$$P_{in} = \frac{V_m I_m}{2} \sin(2\omega t).$$

→ The average power Value is zero for pure Capacitive circuit, because it has double frequency term ($\sin 2\omega t$)

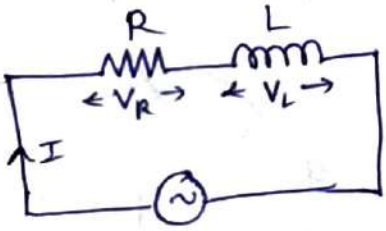
$$\boxed{P_{avg} = 0}$$

Pure capacitor never consumes the power.

Note :- For pure inductive & capacitive circuits never consumes the power, because these are energy stored component circuits.

Analysis of Series RL-Circuit :-

Consider a series RL-Circuit connected with alternating voltage $v = V_m \sin\omega t$ as shown below



In a given circuit, the current 'I' draws by RL-, then it causes two voltage drops as

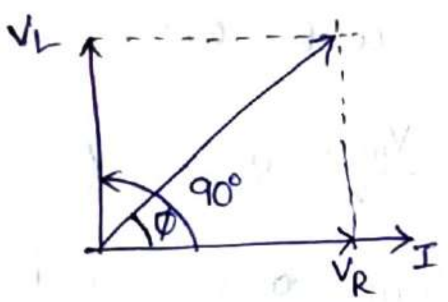
→ Voltage drop across pure resistance $V_R = IR$

→ Voltage drop across pure inductance $V_L = IX_L$

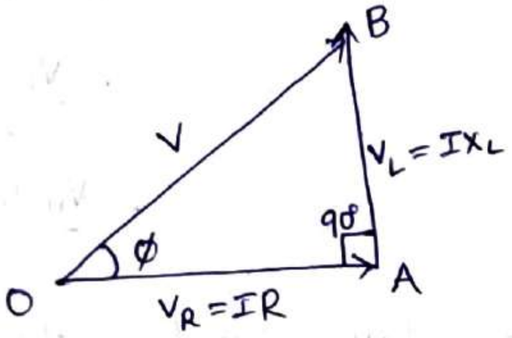
If we will take voltages should be a phasors, then according to KVL, the addition of phasor is

$$\bar{V} = \bar{V}_R + \bar{V}_L = \bar{I}R + \bar{I}X_L$$

by above equation their phasor diagram & Voltage triangle is



phasor diagram



Voltage Triangle

From phasor diagram

$V_R \rightarrow$ is inphase with I

$I \rightarrow$ is lags V_L by 90°

From $\angle OAB$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

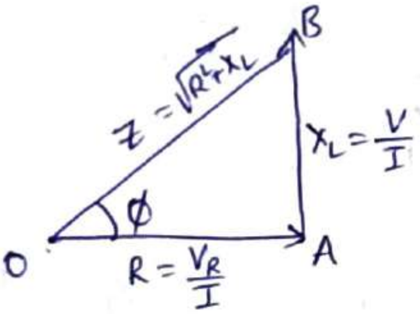
$$V = I.\sqrt{R^2 + X_L^2} = \underline{I.Z}$$

Impedance & Impedance Triangle :-

The impedance for RL-Circuit is $Z = R + jX_L$ (Rectangular form)

$Z = |Z| \angle \phi$ (Polar form)

where $X_L = 2\pi f L$



from $\angle OAB$

$$\tan\phi = \frac{X_L}{R}$$

$$\sin\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{X_L}{Z}$$

→ It can be seen that current lags by voltage by 'ϕ'

so $v(t) = V_m \sin\omega t$

$i(t) = I_m \sin(\omega t - \phi)$

→ The instanteneous Power $P_{inst} = v \times i$

$$= V_m \sin\omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m I_m \sin\omega t \cdot \sin\omega t - \phi$$

$$= \frac{V_m I_m}{2}\left(\cos\phi - \cos(2\omega t - \phi)\right)$$

$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

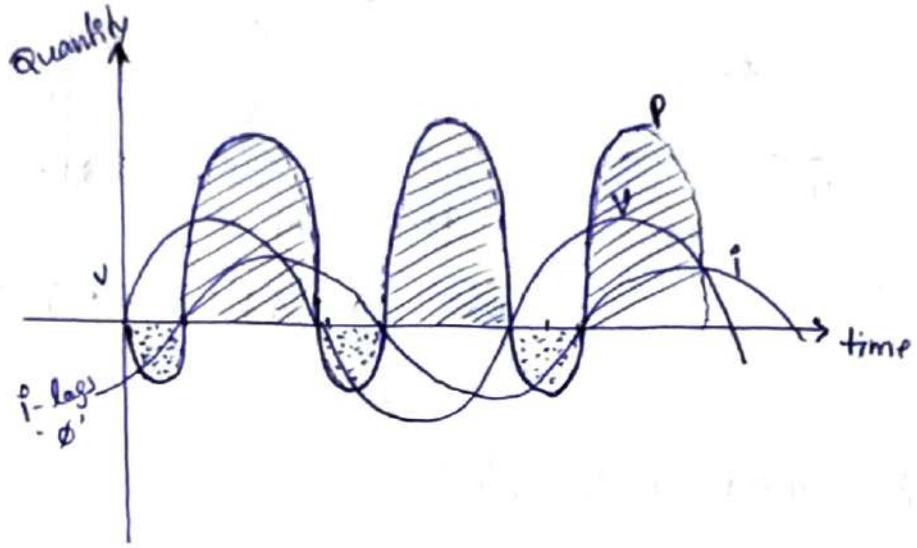
→ In above equation the second term $\cos(2\omega t - \phi)$ has double frequency & average value for cycle is zero

$$P_{avg} = \frac{V_m I_m}{2}\cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

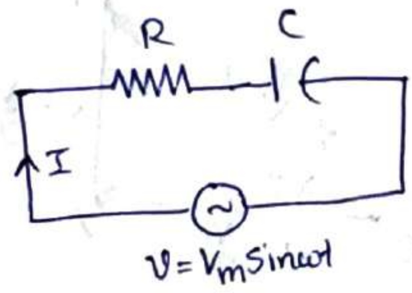
$$\boxed{P = VI\cos\phi} \text{ watts}$$

→ The ~~output~~ waveforms for V, I & power is



Analysis of Series RC-Circuit :

Consider a series RC-Circuit connected with a source $v = V_m \sin\omega t$, as shown below



The give circuit draws the current 'I', causes the two voltage drops

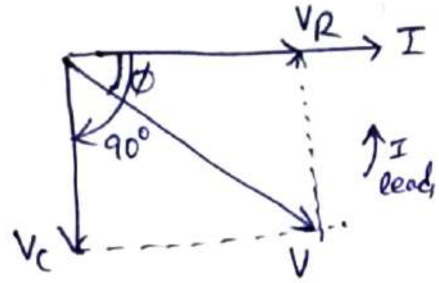
→ Voltage drop across pure resistance $V_R = IR$

→ voltage drop across pure capacitance $V_C = IX_C$

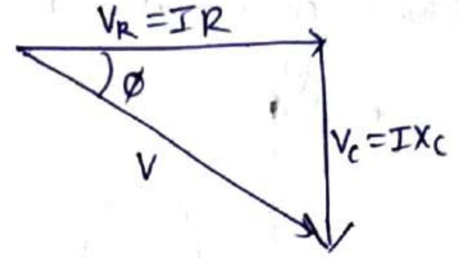
For a given circuit, apply KVL then by addition of two phasors

$$\bar{V} = \bar{V_R} + \bar{V_C} = \overline{IR} + \overline{IX_C}$$

From the above equation their phasor diagram & voltage triangle is



Phasor diagram



From $\angle OAB$

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

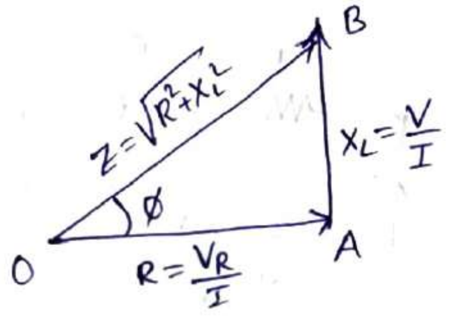
$$\boxed{V = IZ}$$

Impedance & Impedance Triangle :-

The impedance for RC-Circuit is $Z = R - jX_L$ (Rectangul form)

$Z = |Z| \angle -\phi$ (Polar form)

$$X_C = \frac{1}{2\pi f C}$$



from $\angle OAB$

$$\tan\phi = \frac{X_L}{R}$$

$$\sin\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{X_L}{Z}$$

→ It can be seen that current ~~lags~~ leads by voltage 'ϕ'

So $v(t) = V_m \sin\omega t$

$i(t) = I_m (\sin\omega t + \phi)$

→ The instanteneous Power $P_{inst} = v \times i$

$= V_m \sin\omega t \times I_m \sin(\omega t + \phi)$

$= V_m I_m (\sin\omega t \cdot \sin \omega t + \phi)$

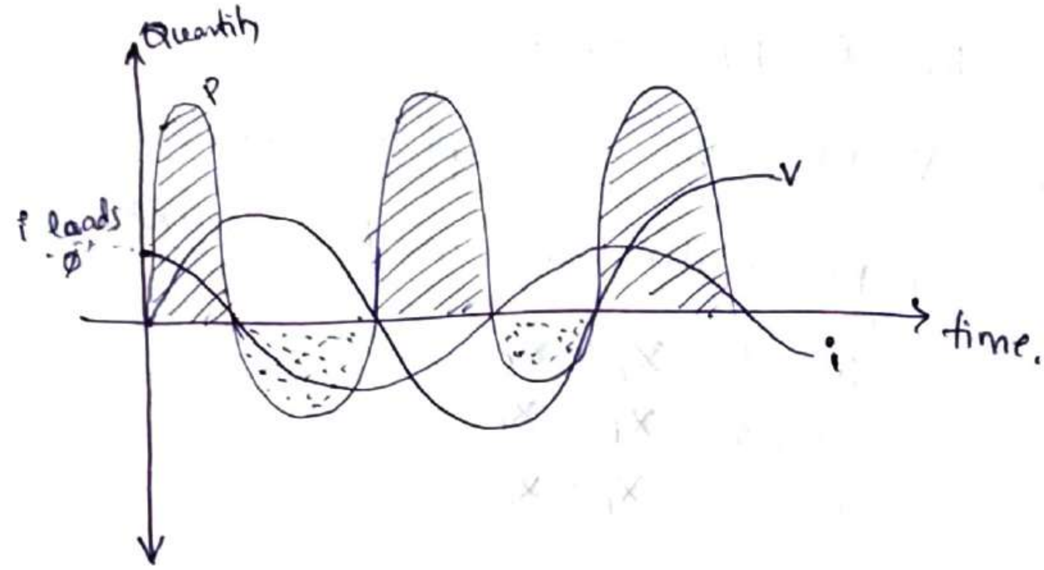
$$P_{inst} = \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t + \phi)$$

In above equation the cosine term "cos2ωt" has double frequency, hence its averge power value is zero, So

$$P_{avg} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\boxed{P_{avg} = VI \cos\phi} \text{ watts}$$

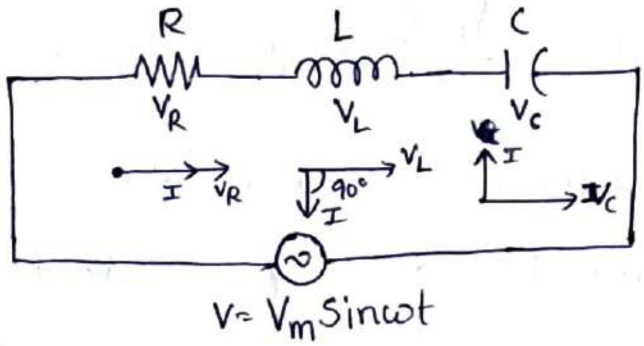
→ Waveforms for V, I & Power is



Analysis of Series RLC Circuit :-

Consider a circuit consisting of resistance R ohms pure inductance L Henries and Capacitance C farads connected in series with supply $v = V_m \sin\omega t$, as shown below.

When supply is connected, then current 'I' draws and it causes 3 - voltage drops



→ Due to current 'I', there are different voltage drops across R, L & C with are given by

→ Drop across resistance R is $V_R = IR$

→ Drop across inductance L is $V_L = IX_L$

→ Drop across Capacitance C is $V_C = IX_C$

By According KVL for RLC-circuit

$$\bar{V} = \bar{V_R} + \bar{V_L} + \bar{V_C}$$ — is phasor addition.

For drawing phasor diagram, we have to consider 3-cases as

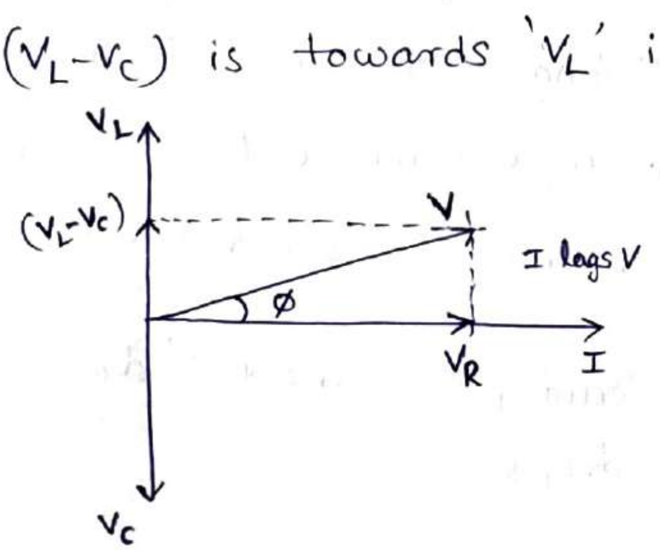
case 1 : $X_L > X_C$

Case 2 : $X_L < X_C$

Case 3 : $X_L = X_C$

Case 1 :- $X_L > X_C$

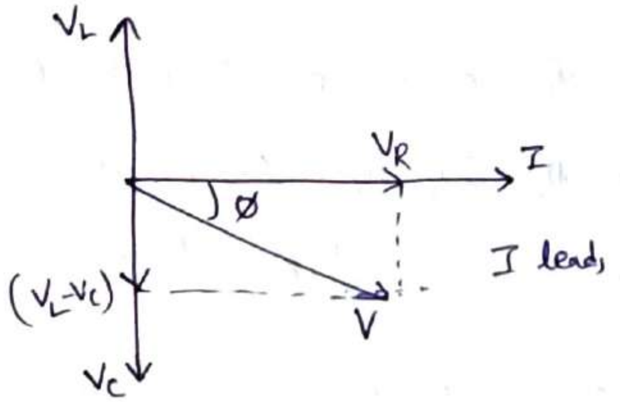
When $X_L > X_C$ then $V_L > V_C$, so resultant of $(V_L - V_C)$ is towards 'V_L' i.e leading I,



→ the circuit behaves as inductive nature

case 2 :- $X_L < X_c$

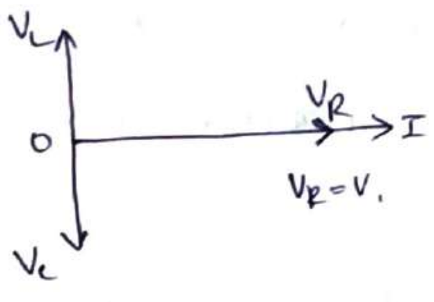
When $X_L < X_c$ then $V_L < V_c$, so resultant of $(V_L - V_c)$ is "$-Ve$" is towards 'V_c' i.e lagging 'I'



i.e the circuit behaves as Capacitive nature.

Case 3 :- $X_L = X_c$

when $X_L = X_c$ then $V_L = V_c$, so resultant of this is zero, means cancel eaach other, $\phi = 0$



i.e the circuit behaves pure resistive nature.

→ This condition $X_L - X_c = 0$ (or) $X_L = X_c$ is called "Resonance condition.

Problems :-

① . A coil has resistance of 4Ω and an inductance of $9.55\ mH$. Calculate (i) the reactance (ii) the impedance (iii) the current taken from a 240V, 50Hz Supply.

② Draw the phasor diagram for a coil of 80mH & 60Ω resistor connected to 200V, 100Hz Supply. Calculate the circuit impedance & the current taken from the supply

③ . A Coils takes a current of 2.5A at 0.8 lagging power factor from a 220V, 60HZ, single phase source. If the coil is modeld by a series RL- Circuit find

(i) the complex power in the coil

(ii) the values of R & L.

④ . Devlop the phasor diagram for a series RC- Circuit with $R = 10\Omega$ & $C = 10\mu F$ & excited with 1-ϕ, 230V Supply.

⑤ . A coil of resistance 5Ω and inductance 120mH in series with a $100\mu F$ capacitor, is connected to 300V, 50Hz supply. Find

(a) The current flowing

(b) The phase difference between Supply voltage & current

(c) The voltage across the coil

(d) The voltage across the capacitor

(e) Draw the phasor diagram.

Resonance in Series RLC-circuit

In a series RLC-circuit, when $X_L = X_C$ ~~(or)~~ $(X_L - X_C) = 0$ i.e when reactance of circuit becomes to zero, then it is called as " the circuit is at "series resonance".

→ At this time circuit behaves a "purely resistive

→ And the power factor $\cos\phi = 1$

→ At this time it draws maximum current.

Applications:-

1. It is used in Tuning circuits
2. In inverters
3. Wave traping circuits.

Resonance frequency (f_r) for Series RLC-Circuit:-

In a ~~series~~ RLC-circuit at which ~~the~~ frequency the resonance occures, that frequency called as "Resonance frequency (f_r).

i.e $$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz.}$$

Let 'f_r' be the frequency at which resonance occurs in series RLC -circuit

→ At resonant frequency 'f_r' the reactance is zero

i.e $X_L = X_C$ or $X_L - X_C = 0$.

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz} \quad \text{or} \quad \boxed{\omega_r = \frac{1}{\sqrt{LC}}} \text{ rad.}$$

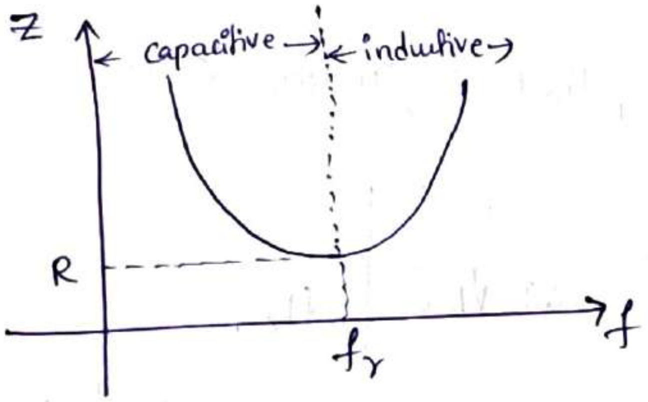
Angular resonance frequency

→ If $f_r > f$ then circuit is Capacitive nature

$f_r < f$ then circuit is inductive nature

$f_r = f$ is pure resistive nature

→ It can be shown as



→ At resonance current is maximum $I_m = \frac{V}{R}$

→ The power at resonance $P_m = I_m^2 R$.

Band width and Quality factor (Q) for Resonance Circuit

In a series RLC-circuit, at resonance condition we have the two half power frequenies which ~~da~~ are due to reactive components, as

$$\left(\omega L - \frac{1}{\omega C}\right) = \pm R$$

Consider two half power frequencies as

$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right) = +R \quad \text{—} (1)$$

$$\left(\omega_2 L - \frac{1}{\omega_2 C}\right) = -R \quad \text{—} (2)$$

→ Add eq (1) + eq (2), then

$$(\omega_1 + \omega_2) L - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = 0$$

$$(\omega_1 + \omega_2) L = \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)\frac{1}{C}$$

$$\omega_1 \omega_2 = \frac{1}{LC} \quad \text{—} (3) \qquad \text{but } \omega_r = \frac{1}{LC}$$

so $\boxed{\omega_1 \omega_2 = \omega_r^2}$ (or) $\boxed{f_1 f_2 = f_r^2}$

→ Substract eq (1) − eq (2), then

$$(\omega_2 - \omega_1) + \frac{(\omega_2 - \omega_1)}{\omega_1 \omega_2} \cdot \frac{1}{LC} = \frac{2R}{L} \qquad \text{multplay } `\tfrac{1}{L}' \text{ on both Side}$$

$$(\omega_2 - \omega_1) + (\omega_2 - \omega_1) = \frac{2R}{L} \qquad \because \omega_1 \omega_2 = \frac{1}{LC}$$

$$(\omega_2 - \omega_1) = \frac{R}{L}$$

$$\boxed{f_2 - f_1 = \frac{R}{2\pi L}}$$

$$\boxed{\text{Band width} = B.W = f_2 - f_1 = \frac{R}{2\pi L}} \quad \& \quad \boxed{\Delta f = \frac{R}{4\pi L}}$$

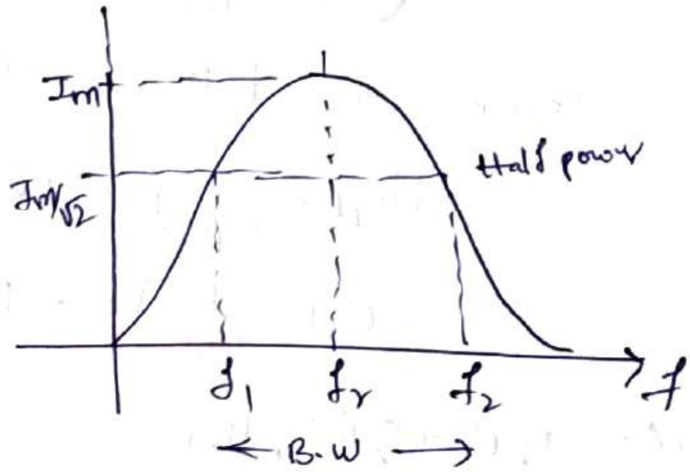
→ And we have

Upper cut-off frequency

$$f_2 = f_r + \Delta f$$

Lower cut-off freq

$$f_1 = f_r - \Delta f$$



Quality factor:-

It ratio of voltage across L (or) C with supply voltage.

i.e voltage magnification is circuit.

$$Q = \frac{V_L}{V} \text{ (or) } \frac{V_C}{V} \text{ (or) } \frac{X_L}{R} \text{ (or) } \frac{X_C}{R}$$

for RL →

$$Q = \frac{\omega_r L}{R} \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

for RC → $\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$ i.e Same.

Parallel RLC Circuits

pro :- In series-parall circuit A & B are in parallel and in series with 'C'. The impedances are

$Z_A = 4+j3\ \Omega$, $Z_B = 4-j5\ \Omega$ & $Z_C = 2+j8\ \Omega$.

If the current $I_c = (25+j0)$, calculate

(i) Branch ~~curr~~ voltage (ii) Branch current (iii) Total power

(iv) Phasor diagram.

Resonance in Parallel circuit :-

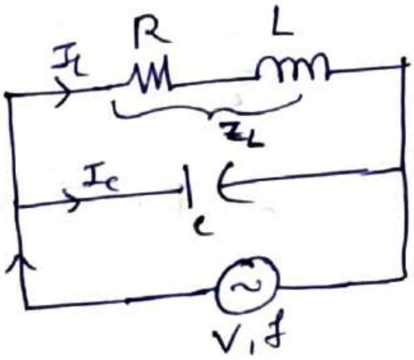
In a parallel circuit when susceptance is becomes to zero, then circuit is said to be in resonance. i.e $(B_L - B_C) = 0$.

→ Power factor is unity. & current is minimum.

→ It is used in Oscillators

→ complex communication circuits & filters.

Consider a parallel circuit which is having series of R&L in parallel with 'c' as shown below



This circuit is called Tuned circuit. If impedance of circuit is Z_L and reactancy are X_L & X_C the at resonance we have relation

$$Z_L^2 = X_L X_C$$

$$R^2 + (2\pi f_r L)^2 = (2\pi f_r L) \times \frac{1}{2\pi f_r C}$$

$$R^2 + (2\pi f_r L)^2 = \frac{L}{C}$$

$$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$2\pi f_r = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

i.f $\frac{R^2}{L^2} << \frac{1}{LC}$ then

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

it is same as ~~resi~~ series resonance

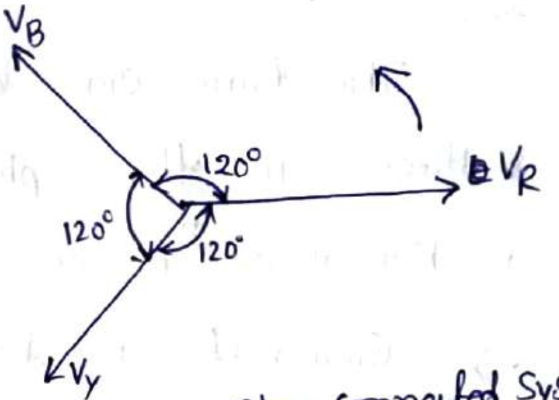
→ Quality factor is current magnification

i.e $$\frac{\frac{V}{Z_L}}{\frac{V}{Z_D}} = \frac{\frac{L}{RC}}{\sqrt{L/C}}$$

then Quality factor $\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$

Three-phase balanced Circuits :-

Three-phase Circuit ~~on~~ System :- The system which is having three voltages with same magnitude & same frequency ~~bem~~ and having each one with a phase difference 120°. As fallow



$$V_R = V_m \sin \omega t$$

$$V_Y = V_m \sin(\omega t - 120°)$$

$$V_B = V_m \sin(\omega t - 240)$$

~~or~~ $V_m \sin(\omega t + 120°)$

→ we have 2-types of 3-ϕ systems
- → Star connected System
- → Delta connected System.

Advatages of 3-ϕ System :-

→ By using 3-ϕ system output power increases, it is 1.5 times of 1-ϕ system power.

→ In 3-ϕ system ~~copper~~ the designing of transmission and distribution it require less copper as comparred with 1-ϕ system.

→ By using 3-ϕ system we can produce rotating magnetic field.

→ In 3-ϕ system instanteneous power is constant

→ Three phase system give steady output.

→ The 3-ϕ system having better regulation, with 1-ϕ System

→ 3-ϕ can be used to supply the 1-ϕ load system

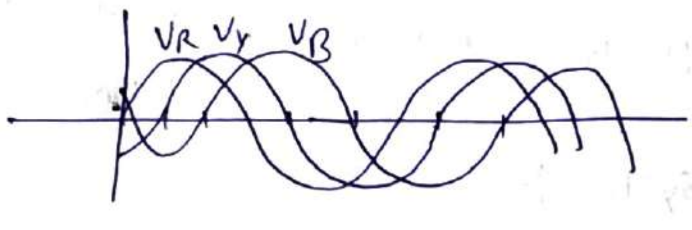
→ 3-ϕ can be rectified into DC-Supply with low ripple factor

→ Parallel operation is easy in 3-ϕ system.

Phase sequence:-

The time order on the sequence in which the voltages in three-phases reach their maximum values is known as "phase sequence".

→ Generally we have R-Y-B is phase sequence



Phase voltage:-(V_{ph}) The voltage which is measured between a line and neutral called as phase voltage

Ex:- $\bar{V}_{RN}$, $\bar{V}_{YN}$ or $\bar{V}_{BN}$

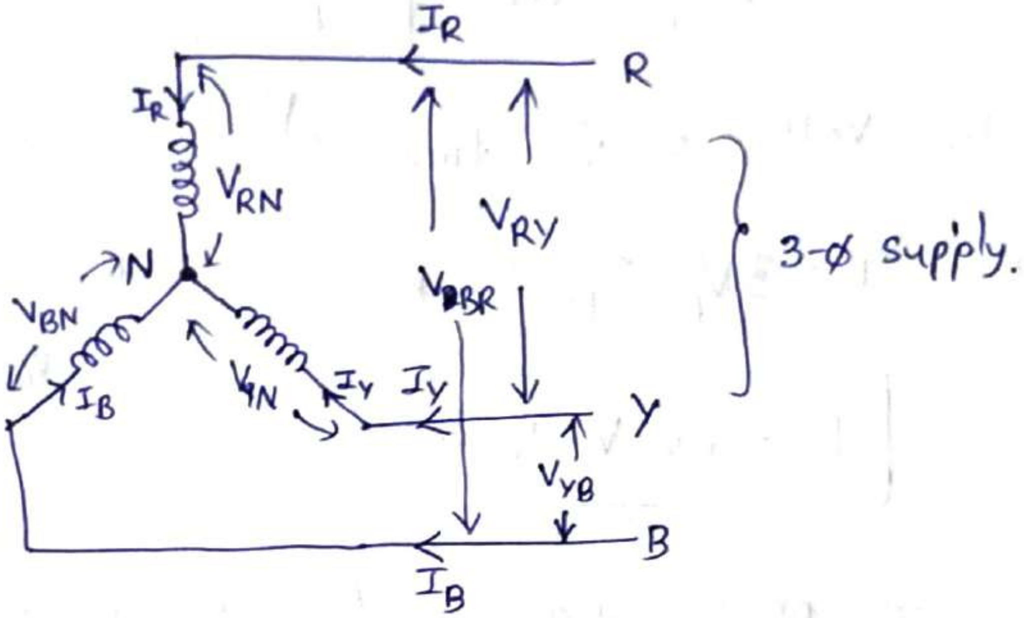
Line voltage:-(V_L): The voltage measured between any two lines in a 3-phase system

Ex: $\bar{V}_{RY}$, $\bar{V}_{YB}$, $\bar{V}_{BR}$

$$\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$$
$$\bar{V}_{YB} = \bar{V}_Y - \bar{V}_B$$

Voltage & Current relations in 3-ϕ Star connected system

The circuit diagram for a 3-ϕ balanced star connected circuit with phase sequence RYB as shown below



For a given 3-ϕ star connected System has

→ Phase voltages $V_{ph} = V_{RN} = V_{YN} = V_{BN}$

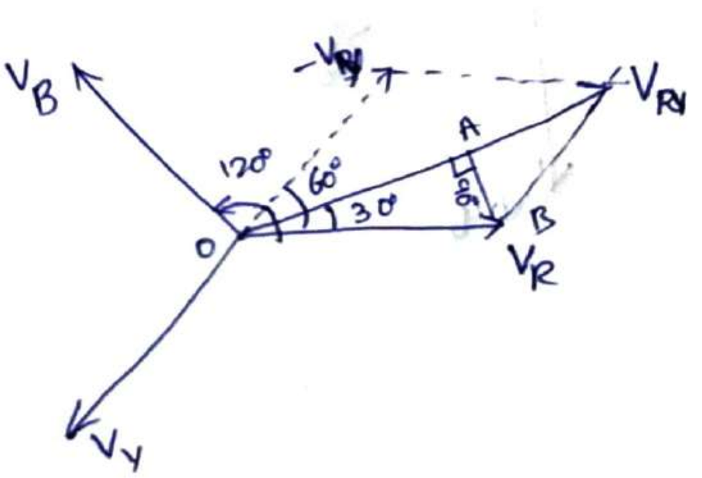
& phase currents $I_{ph} = I_R = I_Y = I_B$

→ Line voltages $V_L = V_{RY} = V_{YB} = V_{BR}$

& line current $I_L = I_R = I_Y = I_B$.

$V_{RY} = V_R - V_Y$
$V_{YB} = V_Y - V_B$
$V_{BR} = V_B - V_R$

→ The phasor diagram for line & phase voltages is



From ΔOAB $\cos 30 = \frac{OC}{OA} \Rightarrow \frac{V_L/2}{V_{ph}} = \frac{\sqrt{3}}{2}$ $\frac{\left(\frac{V_{RY}}{2}\right)}{V_R} = \frac{OC}{OA}$

So, In star connected system

$$\boxed{V_L = \sqrt{3}\ V_{ph}} \quad \text{and} \quad \boxed{I_L = I_{ph}}$$

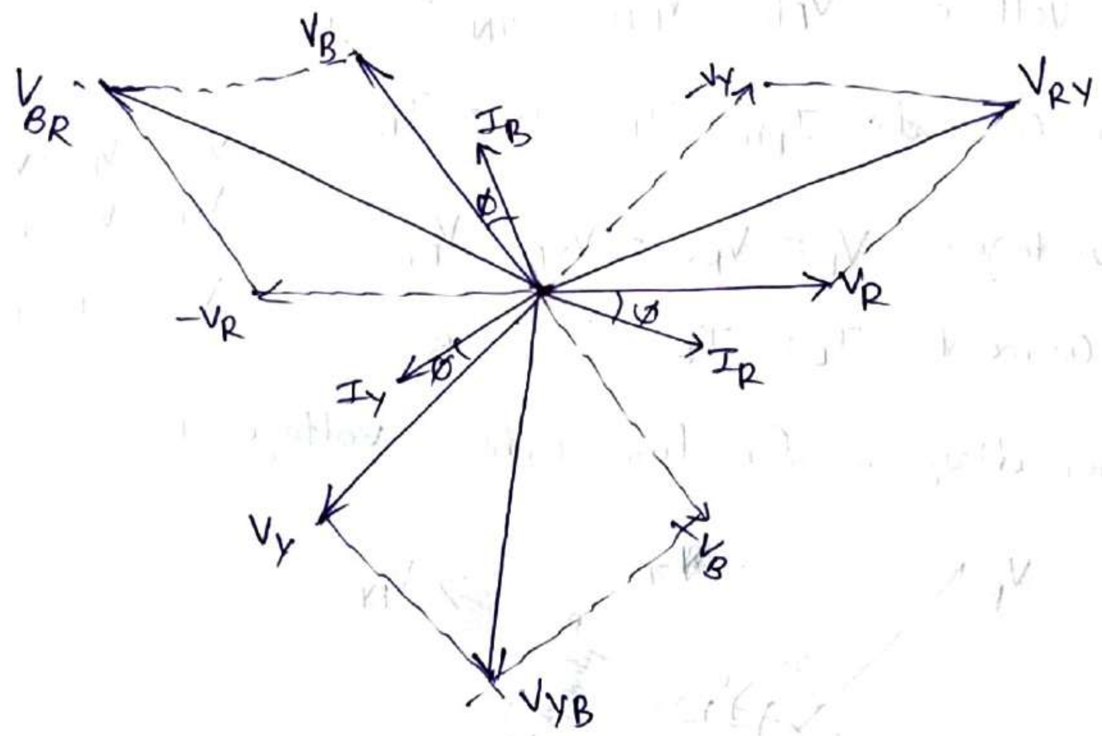
Thuse line voltage is $\sqrt{3}$ times of phase voltage star connection

→ Power $P = 3 V_{ph} I_{ph} \cos\phi$ for 3-ϕ system

$\because V_{ph} = \frac{V_L}{\sqrt{3}}$

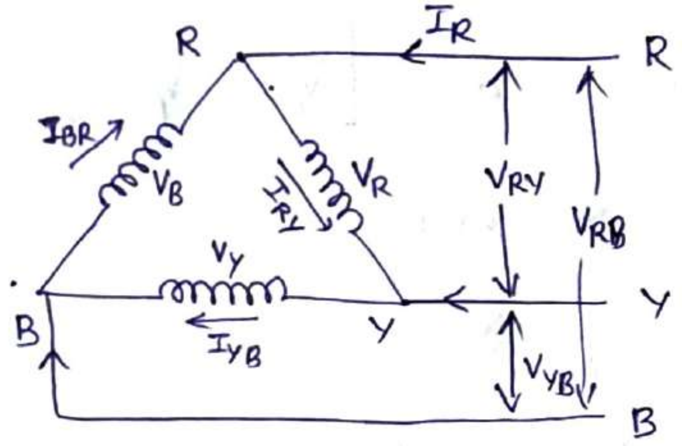
So $$\boxed{P = \sqrt{3}\ V_L I_L \cos\phi}$$

→ The 3-ϕ phasor diagram for star connected inductive load is



Voltage & current relations in 3-ϕ Delta connected System :-

The circuit diagram for a 3-ϕ Delta connected System with phase sequence R-Y-B as shown below.



For a given 3-ϕ delta connected System has

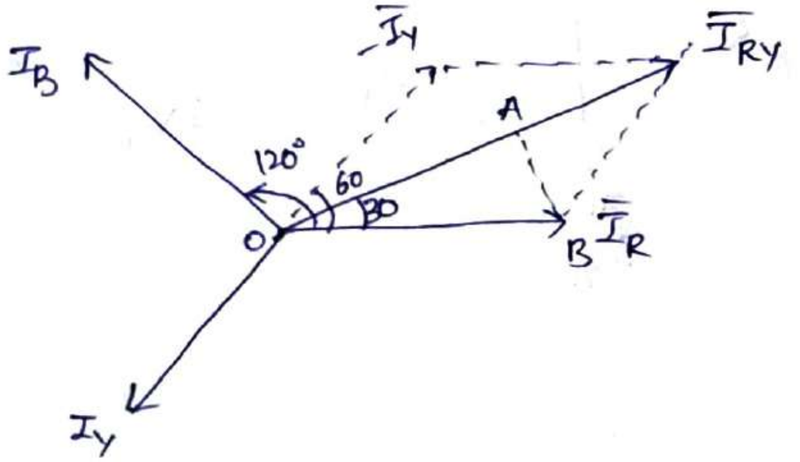
→ phase voltages $V_R = V_Y = V_B = V_{ph}$

& phase currents $I_{ph} = I_{RY} = I_{YB} = I_{BR}$

→ line voltages $V_L = V_{RY} = V_{YB} = V_{BR}$

& line currents $I_L = I_R = I_Y = I_B$

→ The phasor diagram for line current & phase current is



From Triangle OAB $\cos\overset{30}{\phi} = \frac{OC}{OA} = \frac{I_R/2}{I_{RY}}$

$$\frac{\sqrt{3}}{2} = \frac{I_L/2}{I_{ph}}$$

Sor In delta connected System

$\boxed{I_L = \sqrt{3}\ I_{ph}}$ and $\boxed{V_{ph} = V_{\phi L}}$

→ The Power in 3-φ System is

$$P_{ph} = 3V_{ph} I_{ph} \cos\phi$$

$$\boxed{P = \sqrt{3}\ V_L I_L \cos\phi} \text{ watts}$$

→ The 3-φ phasor diagram for delta connected inductive load is